



## ATAR course examination, 2022

### Question/Answer booklet

# MATHEMATICS METHODS

## Section One: Calculator-free

Place one of your candidate identification labels in this box.  
Ensure the label is straight and within the lines of this box.

WA student number: In figures

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In words

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### Time allowed for this section

Reading time before commencing work: five minutes  
Working time: fifty minutes

Number of additional  
answer booklets used  
(if applicable):

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer booklet  
Formula sheet

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	54	35
Section Two: Calculator-assumed	9	9	100	100	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2022: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

**Section One: Calculator-free****35% (54 Marks)**

This section has **six** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

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**See next page**

## Question 1

(9 marks)

Consider the derivative function  $f'(x) = \frac{4x}{x^2 + 3}$ .

(a) Determine the rate of change of  $f'(x)$  when  $x = 1$ .

(3 marks)

(b) Determine  $f(x)$  given that  $f(1) = \ln(32)$ .

(4 marks)

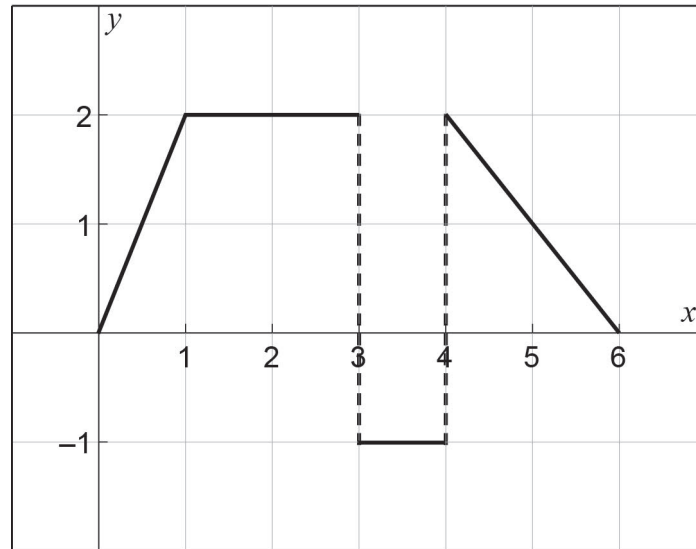
(c) Determine  $\frac{d}{dt} \int_t^3 f(x) dx$ .

(2 marks)

## Question 2

(6 marks)

Consider the function  $f(x)$  shown below.



Evaluate the following integrals.

(a)  $\int_0^6 f(x) dx$  (2 marks)

(b)  $\int_0^4 f(x) - 2 dx$  (2 marks)

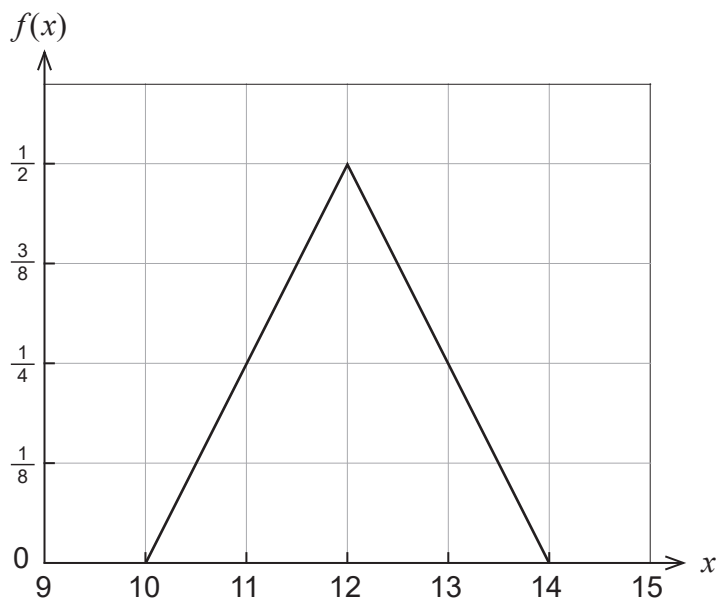
(c)  $\int_4^6 f'(x) dx$  (2 marks)

See next page

Question 3

(11 marks)

Arnold would like to purchase a toy for his child’s birthday. The Isosceles Toy Company claims that the number of weeks until delivery,  $X$ , is a random variable whose probability density function is displayed in the graph below.



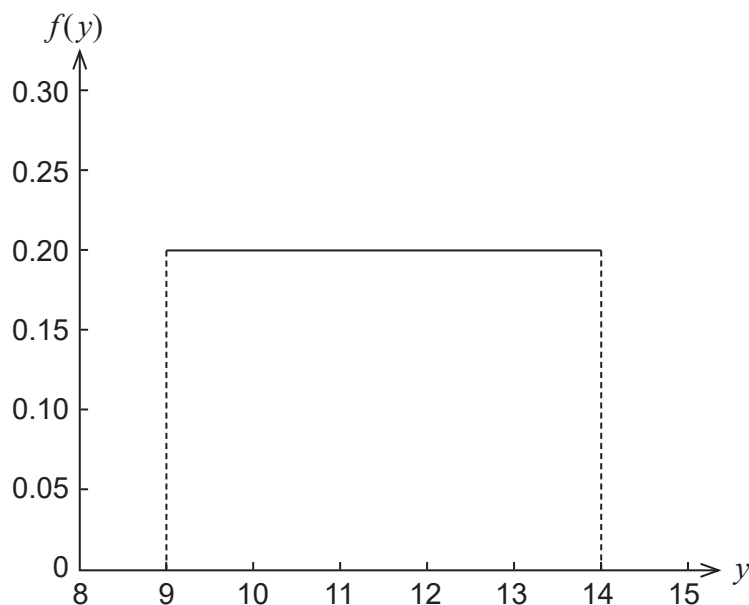
- (a) What is the expected time for the toy to be delivered? (1 mark)

His child’s birthday is 13 weeks away.

- (b) What is the probability that the Isosceles Toy Company will deliver the toy in time for his child’s birthday? (2 marks)

- (c) Given that the toy arrives in time for his child’s birthday, what is the probability that it arrives at least one week early? (2 marks)

Uniform Toys, a rival toy company, claims that the number of weeks until delivery of the same toy,  $Y$ , is a random variable whose distribution is displayed in the graph below.



- (d) Which toy company should Arnold choose if he would like to maximise the chance that the toy will be delivered in time for his child's birthday? Why? (2 marks)

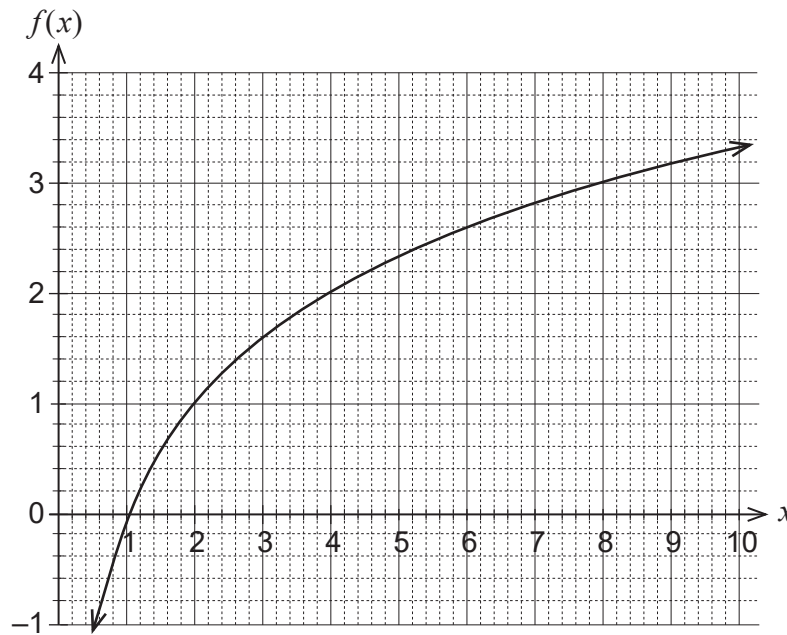
Suppose that five people order the toy from Uniform Toys and let  $Z$  be a random variable that denotes the number of those people who receive the toy within 13 weeks.

- (e) State the distribution for  $Z$ . (2 marks)
- (f) What is the probability that four out of the five people receive the toy within 13 weeks? (2 marks)

Question 4

(12 marks)

The graph of the function  $f(x) = \log_2(x)$  is shown below.



(a) Using the graph:

(i) solve  $\log_2(x - 5) = 3$ .

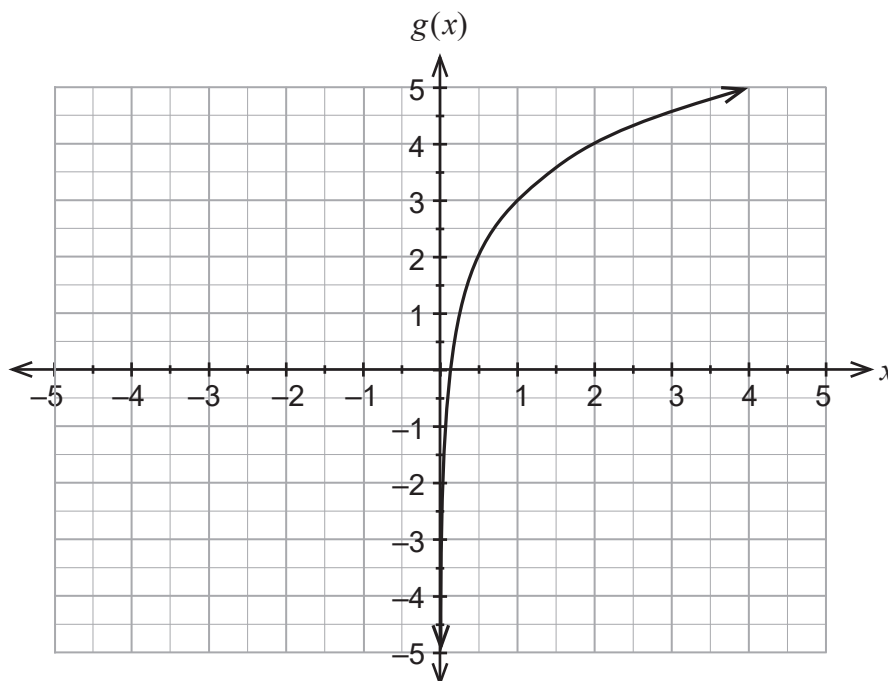
(2 marks)

(ii) determine  $\sqrt{7}$ , correct to one decimal place. (Hint: let  $x = \sqrt{7}$ .)

(3 marks)



- (b) The function  $f(x) = \log_2(x)$  is translated to give the new function  $g(x)$ , which is shown in the graph below.



Determine the equation for  $g(x)$ .

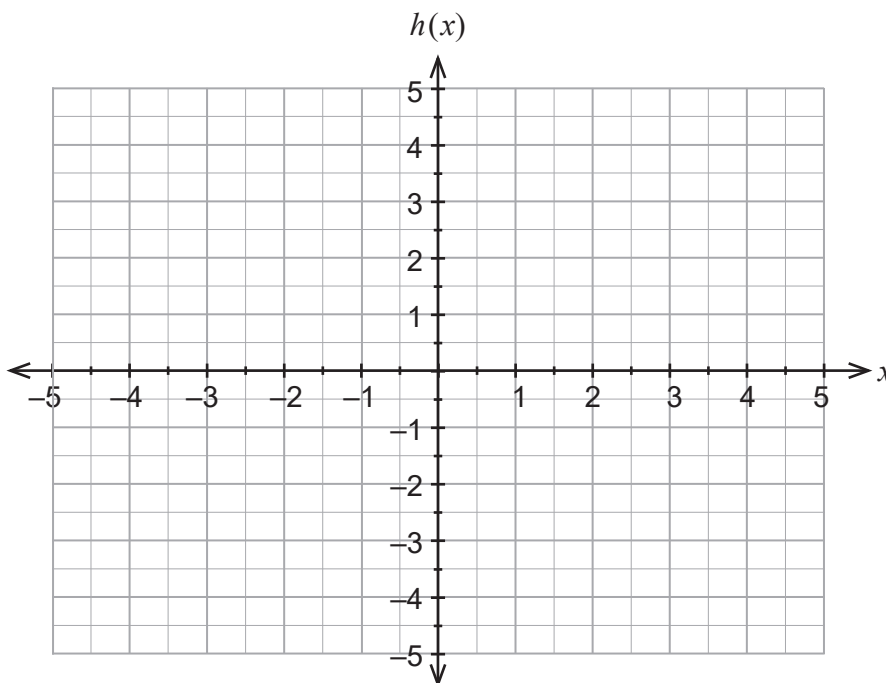
(2 marks)

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Question 4 (continued)

- (c) (i) Show that  $\log_2\left(\frac{1}{x-1}\right) = -\log_2(x-1)$ . (2 marks)

- (ii) Hence sketch the graph of  $h(x) = \log_2\left(\frac{1}{x-1}\right)$  on the axes below. (3 marks)



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

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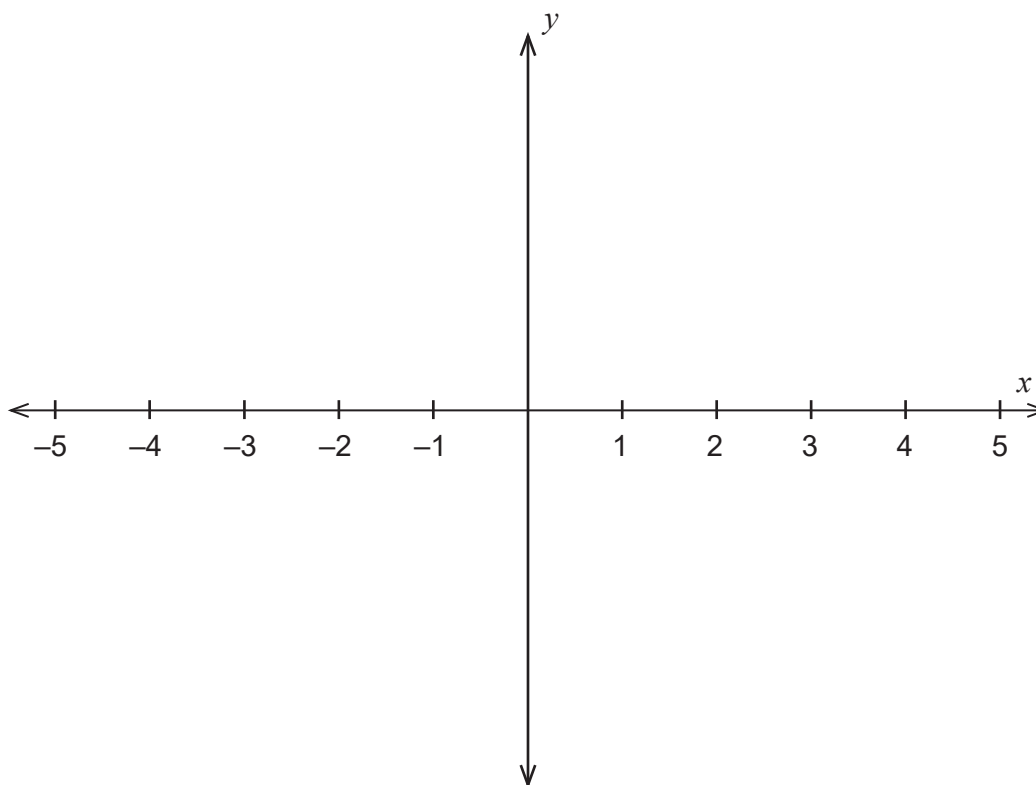
**Question 5**

**(5 marks)**

A continuous function,  $f$ , satisfies the following conditions:

- $f(2) = 0$
- $f$  has exactly 2 stationary points
- $f'(-1) = 0$  and  $f'(1) = 0$
- $f''(-1) = 4$
- $f'(2) > 0$ .

Sketch the function on the axes below.



A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

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Question 6

(11 marks)

The table of values below may be used to assist you in answering part (b) of this question.

$\sin(0) = 0$	$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\sin\left(\frac{\pi}{2}\right) = 1$
$\cos(0) = 1$	$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\cos\left(\frac{\pi}{2}\right) = 0$

(a) (i) Determine  $\frac{d}{dx}\left(x \sin\left(\frac{\pi x}{4}\right)\right)$ . (2 marks)

(ii) Hence show that

$$\int \frac{\pi x}{4} \cos\left(\frac{\pi x}{4}\right) dx = x \sin\left(\frac{\pi x}{4}\right) + \frac{4}{\pi} \cos\left(\frac{\pi x}{4}\right) + c$$

where  $c$  is a constant. (3 marks)

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- (b) The time in minutes,  $T$ , between incoming phone calls at a call centre is a random variable with probability density function

$$p(t) = \begin{cases} \frac{\pi}{4} \cos\left(\frac{\pi t}{4}\right), & 0 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Determine the probability that the time between two consecutive phone calls is less than 40 seconds. State your answer exactly. (3 marks)
- (ii) Use the result from part (a)(ii) to determine the expected time between consecutive phone calls. (3 marks)

Supplementary page

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Supplementary page

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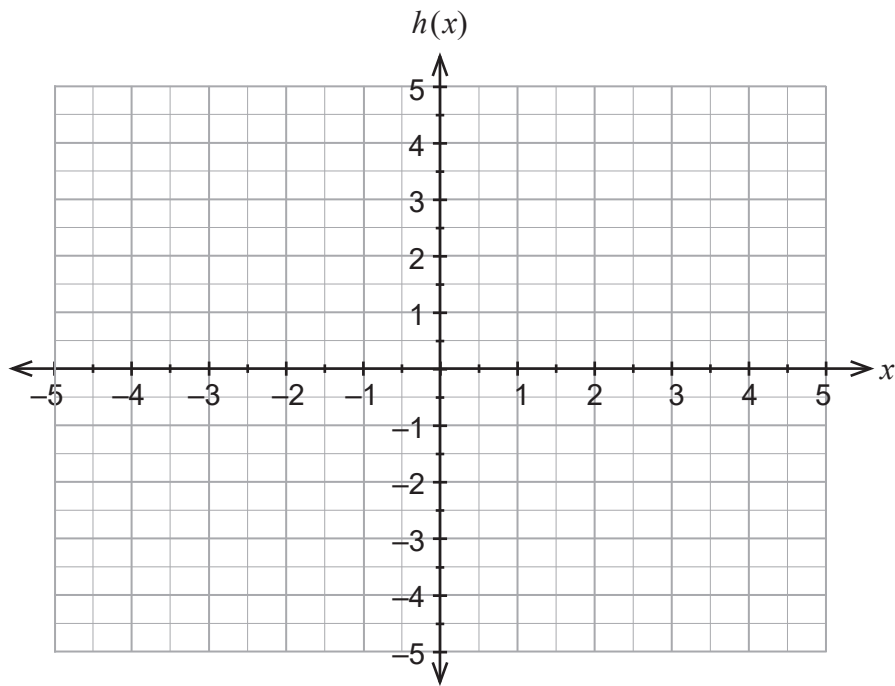


Supplementary page

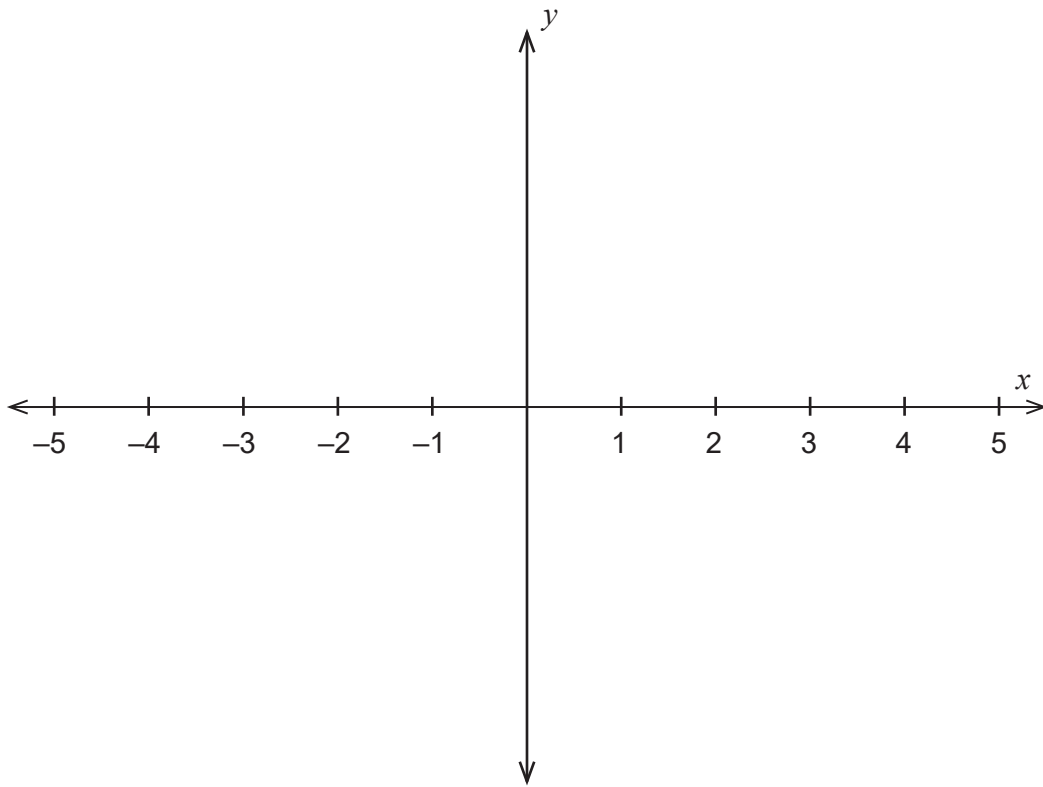
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Spare grid for Question 4(c)(ii)



Spare grid for Question 5



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